DOCUMENT RESUME

ED 031 096 88 EM 007 326

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A Beta Index to Confirm Causal Directions in a Closed System of Five Variables.

Spons Agency-Office of Education (DHEW), Washington, D.C.
Report No-ESEA-3; OE-BESE
Pub Date 7 Feb 69
Note-47p.; Paper presented at American Educational Research Association, Los Angeles, Calif., Feb. 7, 1969
EDRS Price MF-$0.25 HC-$2.45
Descriptors-Experiments, Logic, Mathematical Concepts, *Mathematical Models, *Mathematics, *Relationship, *Research

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ED031096

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A BETA INDEX

TO CONFIRM CAUSAL DIRECTIONS

IN A CLOSED SYSTEM OF FIVE VARIABLES

By

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Presented in the Experimental Session: Experimental Design and Analysis
Annual Meeting of the American Educational Research Association
Paper Session 12.12, Division D
Los Angeles, California
February 7, 1969

EM007326

A BETA INDEX TO CONFIRM CAUSAL DIRECTIONS IN A CLOSED SYSTEM OF FIVE VARIABLES

ABSTRACT

The beta coefficient of an intermediate variable in a causal direction remains relatively constant as other system variables are introduced and controlled in stepped regression, whereas that in the acausal direction changes noticeably--the underlying intuitive principle of the study. The main strategy was to compare changes in beta coefficients of the intermediate variable from pairs of stepped equations in the causal direction to changes in the acausal direction of 3-variable paths.

200 x 5 normalized random numbers were generated and substituted in interdependent equations to produce 5 scores for each of 200 pseudo-people. Stepped-regression analysis was then applied on all possible 3-variable paths. Beta differentials on a given intermediate variable were computed and compared for opposite directions; the smaller of the two was treated as that of the causal direction and became the numerator of a beta index, and the larger became the denominator. The index persisted to obtain values between 0 and +1 in the one direction as other system variables were stepped in; the closer to 0, the stronger the causal direction.

But some remaining 3-variable paths imitated the known causal paths. Hence, further study was conducted to find support for the known causal paths and to eliminate the imitations. Comparisons of beta coefficients for end variables in regression equations and comparisons of correlations among the 3 path variables of known causal paths yielded mathematically-consistent like orders of magnitude as the beta differentials.

It was concluded that the beta index, found wanting initially, had greater utility as a confirming device by the addition of the 2 modifications in that all but the originally designed causal paths were eliminated as lacking causal-direction consistency. But obviously the index procedure might well be confined to the computer presently.

A BETA INDEX TO CONFIRM CAUSAL DIRECTIONS IN A CLOSED SYSTEM OF FIVE VARIABLES

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The researcher often has difficulty convincing himself and others about his findings on causal paths within a system of variables, regardless of rational conclusions, for he may not have included all relevant variables. Blalock, in pulling together materials on causal models, attempted to spell out the rules of causal inference, thus shortening the gap between theory and reality. His main strategy was to study beta coefficients in regression equations; such strategy served as the springboard for the present study.

In the exploratory construction of a Blalockian causal model, it was discovered that the beta coefficient of an intermediate variable in a causal direction remains relatively constant as other system variables are introduced and controlled in stepped regression, whereas that in the acausal direction changes noticeably--the beta constancy principle, upon which a beta index was developed as illustrated in Figure 1. For clearcut three-variable causal paths, the inequality between the magnitudes of the two beta differentials persisted, as remaining variables were stepped in, apparently a case of asymmetry emphasized by Blalock (p. 36); that is, the beta differential on the intermediate variable was less in the causal direction, or

$$d\beta_c < d\beta_a \qquad (1)$$

But for other paths of connected variables, the inequality displayed inconsistencies such as reversals. The two members of the inequality were summarized in the form of a ration, called the beta index, such that the smaller of the two, treated as the beta differential for the causal direction, became the numerator. The value of this index, hence, was made to vary between +1 and 0; the closer to 0, the stronger the causal direction. From Inequality (1), therefore,

$$I_\beta = \frac{d\beta_c}{d\beta_a} \qquad (2)$$

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Insert Figure 1 about here

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The problem then evolved as to whether the beta index could be demonstrated in a closed system of variables derived from dummy data and related by known causal paths by a system of equations.

METHOD

200 x 5 normalized random numbers (R_i) were generated[2] and substituted into Equations 3 through 7 to obtain 5 scores (X_i) for each of 200 pseudo-people. These equations form the model portrayed in Figure 2, causal

directions indicated by arrows[3].

$$X_1 = R_1 \tag{3}$$

$$X_2 = R_2 \tag{4}$$

$$X_3 = b_{31}X_1 + b_{32}X_2 + R_3 \tag{5}$$

$$X_4 = b_{41}X_1 + b_{42}X_2 + R_4 \tag{6}$$

$$X_5 = b_{52}X_2 + b_{54}X_4 + R_5 \tag{7}$$

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Insert Figure 2 about here

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The b weights were set equal to 1 to find X_3, X_4, and X_5, to simplify manipulation and analysis for the purpose of the present study. X_i then served as input in stepped multiple-regression analysis, the output consisting of the required beta coefficients and other ancillary data.

In the initial stages of data processing, certain checks were conducted: (1) the 5 x 5 R_i correlation matrix revealed no r significantly different from 0 at the .01 level (see Table A); (2) the left and right members of Equations 3 through 7 were found equal, to five significant figures, when appropriate mean R_i and mean X_i were substituted (see Table B); and (3) the b weights found in regression equations approximated the b weights in Equations 3 through 7, with predicted X_i and mean X_i being found equal, to five significant figures (see Table C).

Following rules outlined by Blalock (pp. 61-94), the X_i correlations produced other checks to confirm the efficacy of the model, as outlined in Table 1[4]. These final checks thus allowed further study of beta weights.

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Insert Table 1 about here

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Directions of the arrows in Figure 2 were disregarded initially in establishing possible causal directions in the three-variable paths. Nine paths were obvious: 132, 142, 145, 245, 254, 314, 324, 325, and 425, each clearly bi-directional[5]. In the analysis, however, paths 145 and 245 were studied first, since they were the only two originally designed paths with causally connected (initial-cause, intermediate, and final-effect) variables.

The basic procedure included the following steps.

(1) A path of three variables was selected, and a causal direction was tentatively established, to include initial-cause, intermediate, and final-effect variables.

(2) A pair of regression equations was found for the final-effect variable, predicted first by the intermediate variable alone and second with the initial-cause variable stepped in. The beta differential on the intermediate variable was then computed ($d\beta_c$). In like manner, the beta

differential was computed for the acausal direction ($d\beta_a$); that is, the initial-cause variable was predicted. The magnitudes of the two differentials were compared for order of value.

(3) Step 2 was repeated, each time with one more of the remaining system variables stepped in and controlled. The end variable, initial-cause or final-effect, was always stepped in last to find the beta differential on the intermediate variable, such that the newly introduced variable was reflected in each of the pairs of equations. The beta differential was always taken on the original intermediate variable.

(4) The decision was then made to retain or reject the causal direction established in Step 1, pending further study of imitations of known causal paths. If all comparisons of beta differentials from equation pairs in Steps 2 and 3 yielded the same order of value, the decision was made to retain.

The basic procedure was then switched to exploratory study of the imitations of known causal paths, such that (1) the magnitudes of end-variable beta coefficients were studied and (2) the magnitudes of the three correlation coefficients of the three-variable paths were studied. That is, a search was made for mathematical and logical consistencies.

The total variances in predicted variables X_3, X_4, and X_5 were then checked for contribution by component endogenous X_i and exogenous R_i.

Finally, in order to find explanation for apparent mathematical consistencies, the algebraic relations between beta and correlation coefficients were studied.

RESULTS

The study of beta differentials on intermediate variable X_4 of path 145 is summarized in Table 2. First, comparison of β_{14} and β_{54}, opposite directions, yielded nothing on which to base a decision as to causal direction. After the third (end) variable was stepped in for each direction, it was again noted that a comparison of betas, $\beta_{14.5}$ and $\beta_{54.1}$, yielded nothing for decision, and, in fact, the two sets of betas in predicting final variables were reversed in magnitude of values. However, the beta changes resulted in beta differentials which apparently supported the beta constancy principle. Obviously, the difference between β_{14} and $\beta_{14.5}$, $d\beta_{14}$ for the acausal direction, was found to be greater than the difference between β_{54} and $\beta_{54.1}$, $d\beta_{54}$ for the causal direction. The change in slope (beta) of the intermediate variable was apparently relatively constant in predicting variable X_5 (actually Z_{X_5}), but it was noticeably greater in predicting variable X_1. Tentatively, the causal direction of path 145 was accepted as 1-4-5 rather than 5-4-1 (see Footnote 5). Since direction 1-4-5 was designed as causal, the decision appeared valid.

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Insert Table 2 about here

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When the fourth and fifth variables were stepped into regression and controlled, similar results were obtained. That is, the magnitude of $d\beta_{14}$ was found to be greater than $d\beta_{54}$ at each step, the intermediate variable being X_4 and system variables X_2 and X_3 being introduced and controlled. It is emphasized that end variable X_5 was stepped in last in predicting X_1 for each equation pair; and X_1, last in predicting X_5.

The study of beta differentials on intermediate variable X_4 of path 245 resulted in similar directional patterns as for path 145. At each step of analysis, the magnitude of $d\beta_{24}$ was greater than that of $d\beta_{54}$. Hence, the causal direction 2-4-5 was accepted (see Table 3).

What was expected for paths 145 and 245 as to directionality was actually found, that the beta coefficient on the intermediate variable in predicting the final-effect variable remained relatively constant while that in predicting the initial-cause variable (the opposite direction) changed noticeably.

The other seven paths were then analyzed, pertinent findings reported in Table 3, as derived from appropriate beta coefficients (see Table D). It was found that five directions, 2-3-1, 2-4-1, 2-5-4, 3-2-5, and 4-2-5, could also be causal based on beta-differential comparisons for all steps; that is, they imitated directions 1-4-5 and 2-4-5. Although directions 3-1-4 and 3-2-4 could be causal on the first two steps, they reversed on the last step of analysis (see Table 3), and hence, paths 314 and 324 were eliminated. From Figure 2, however, findings about the imitators could not be logical.

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Insert Table 3 about here

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Exploratory study of regression equations revealed some amplifying information about directions 2-5-4 and 4-2-5 which was similar to that about directions 1-4-5 and 2-4-5. The beta differentials for end variables, initial-cause and final-effect, compared the same as the beta differentials in like directions. For example, the magnitude of β_{51} was less than β_{15} for path 145 (see Table 2), at each step for all steps, just as d_{54} was less than d_{14}, at each step for all steps. But obviously, directions 2-5-4 and 4-2-5 were not causal in design, and further differentiations would have to be made to eliminate them as causal. However, directions 2-3-1, 2-4-1, and 3-2-5 did not yield the same beta-coefficient patterns for end variables--and later they were found mathematically inconsistent--and were, therefore, not accepted as causal.

Because both zero-order and partial correlations could be misleading, the rationale of Blalock (pp. 65-77) was resorted to in analyzing the directions 1-4-5, 2-4-5, 2-5-4, and 4-2-5. The basic observation which Blalock (p. 73) attributes to others was then checked, that the magnitude of correlations between variables farthest apart in causal sequence should be smallest. At the same time, it was reasoned that the most recent relationship in the causal sequence should be greater in magnitude than any causally-prior relationship. (Together, these two points seemed to summarize the relationships among grandfather, father, and son over time.) In Table 4, zero-order correlations were first recorded, then followed by partials controlling for the effects of causally-prior variables having a direct effect on any of the two correlated variables. For the originally designed causal directions, 1-4-5 and 2-4-5,

it could be observed that the correlations between the initial-cause and final-effect variables, between the initial-cause and intermediate variables, and between the intermediate and final-effect variables were ascending in order of magnitude. For the remaining directions, 2-5-4 and 4-2-5, such order did not hold and hence they were rejected.

Insert Table 4 about here

The basic four-step procedure in finding beta differentials on which to base decisions as to causal directions seemed useful only for confirming but not for determining such directions, because many imitators were found. The refinements through study of end-variable beta coefficients and path correlations could conceivably strengthen the usefulness of the procedure, as supported mathematically in the discussion section.

As a confirming device, the beta index demonstrates that directions 1-4-5 and 2-4-5 are causal, as distinct from the acausal directions, in paths 145 and 245, respectively. Its utility was in question, however, despite its appeal to intuition, because of imitators found. Clearly then, if mathematical patterns could be determined beyond the basic four steps into exploratory study, and consistencies also found, persistent beta indices would have greater utility as confirming devices for causal directions in a closed system of variables, the model for which could be synthesized in different ways such as via Blalock.

DISCUSSION

Mathematics of Beta Differentials

Examination of causal direction 1-4-5 revealed the relationships in Inequalities 8, 9, and 10, the reduction of the beta-differential comparison to correlation comparison.

$$|\beta_{54} - \beta_{54\cdot 1}| < |\beta_{14} - \beta_{14\cdot 5}| \tag{8}$$

$$\left| r_{45} - \frac{r_{45} - r_{14}r_{15}}{1 - r_{14}^2} \right| < \left| r_{14} - \frac{r_{14} - r_{15}r_{45}}{1 - r_{45}^2} \right| \tag{9}$$

$$\left| \frac{r_{14}}{1 - r_{14}^2} \right| < \left| \frac{r_{45}}{1 - r_{45}^2} \right| \tag{10}$$

Obviously, the most recent relationship in the causal sequence is greater in magnitude than the causally-prior relationship. Hence, the relationship

$$|r_{14}| < |r_{45}| \tag{10a}$$

must be true. The denominator of either member of Inequality 10, disregarding absolute-value symbols, serves to increase the value of the term curvilinearly, where r of ± 1 are the asymptotes; that is, the size of the correlation apparently increases (see Figure A).

Logically then, such a simple relationship as Inequality 10 could be very useful when causal relationships are initially studied; that is, by merely comparing correlations in a possible causal direction for three-variable paths. For the beta-differential comparisons to have any meaning, however, both Inequalities 8 and 10 must be true. But Inequality 10a must still be searched for.

When the fourth and fifth variables are reflected in beta-differential studies, the algebraic manipulations become extremely complicated to the point of becoming meaningless. However, geometric trace lines of hyperplanes may be pictured in the same manner as those in Figure 1. Reduction to simple correlations of higher-order beta coefficients is difficult, but this fact enhances the utility of beta differentials.

Mathematics of End-Variable Beta Coefficients

Examination of causal direction 1-4-5 revealed further relationships in Inequalities 11 through 18.

$$|\beta_{51\cdot 4}| < |\beta_{15\cdot 4}| \qquad (11)$$

which becomes

$$\left|\frac{r_{15} - r_{14}r_{45}}{1 - r_{14}^2}\right| < \left|\frac{r_{15} - r_{14}r_{45}}{1 - r_{45}^2}\right|$$

which simplifies to

$$|r_{14}^2| < |r_{45}^2|$$

Hence,

$$|r_{14}| < |r_{45}| \qquad (12)$$

which is mathematically consistent with Inequality 10 and which now can be considered true. The end-variable beta coefficients must compare in order of magnitude the same as the beta differentials in like directions. At the same time, the most recent relationship in the causal sequence should be greater in magnitude than the causally-prior relationship.

In like manner, if

$$|\beta_{51\cdot 24}| < |\beta_{15\cdot 24}| \qquad (13)$$

then

$$|r_{14\cdot 2}| < |r_{45\cdot 2}| \qquad (14)$$

If

$$|\beta_{51\cdot 34}| < |\beta_{15\cdot 34}| \qquad (15)$$

then

$$|r_{14\cdot 3}| < |r_{45\cdot 3}| \qquad (16)$$

If

$$|\beta_{51\cdot234}| < |\beta_{15\cdot234}| \tag{17}$$

then

$$|r_{14\cdot23}| < |r_{45\cdot23}| \tag{18}$$

Obviously, the pattern in Inequalities 11 through 18 are all consistent with that in Inequalities 8 through 10. All values for these relationships were found for path 145.

All values for corresponding relationships were also found for path 245, the other causal path (see Tables D and E).

Extending the pattern of correlations of Inequalities 14, 16, and 18, it should follow that

$$|r_{14\cdot235}| < |r_{45\cdot231}| \tag{19}$$

And then,

$$|r_{14\cdot25}| < |r_{45\cdot21}| \tag{20}$$

$$|r_{14\cdot35}| < |r_{45\cdot31}| \tag{21}$$

and

$$|r_{14\cdot5}| < |r_{45\cdot1}| \tag{22}$$

Inequalities 19 through 22, in that they resemble and are consistent with Inequality 12, seemingly correspond to higher-order beta differentials much like Inequality 8 was simplified to Inequality 10. The mathematical relationships were not checked because of complicated intricacies, but nevertheless, the values for these relationships were found for path 145.

All values for corresponding relationships were also found for path 245, as indicated by Inequalities 19 through 22 for path 145 (see Table E).

All relationships for path 145 expressed in Inequalities 8 through 22 were found as well as correspondingly for path 245. And too, with the relationships found in Table 4, one becomes amazed at the consistencies found for the causal directions as designed, whereas the other three-variable paths display inconsistent patterns (see Table F). The patterns found that must be true are generalized in Table 5.

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Insert Table 5 about here

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Limitations of Mathematical Relationships

Although all mathematical relationships outlined hold for the designed causal directions 1-4-5 and 2-4-5, and although they apparently extend the beta constancy principle, they seem to detract from the original intent of demonstrating the beta index in confirming causal directions. But the sidestep was exploratory. Meanwhile with the world not so neatly designed, caveat emptor.

Total Variances of Predicted Variances

Total variances for predicted variables X_3, X_4, and X_5 are accounted for by both X_i and R_i (see Table 6). Obviously, 100% of the variance in each cannot be accounted for within the closed system of X_i alone. However, when appropriate R_i are stepped in, 100% of the variance of each predicted variable is achieved. Since the basic model of X_i in Figure 2 is less than ideal[6], the introduction of outside variables with X_i could change the beta-index concept, also a subject for later study. However, since most of the variance of each of the predicted variables at the final-effect end of a causal direction is accounted for within the system of X_i, use of the beta index to confirm causal directions in exploratory models is considered adequate presently.

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Insert Table 6 about here

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Modifications

The four-step procedure for comparing beta differentials was repeated in two limited side studies: (1) with a horizontal distribution of R_i and (2) with randomized b coefficients in Equations 3 through 7. In the first, relationships in paths 145 and 245 were again found; whereas in the second, relationships in paths 145 and 245 were found inconsistent for many inequalities with beta indices approaching +1. It appeared that in the second study a symetry of beta differentials about the intermediate variable X_4 was achieved, and the model, as modified, could not be causal at all. It appeared that further study of the causal model with a horizontal distribution of R_i could open other avenues for exploration of the utility of the beta index.

CONCLUSIONS

The problem of using a beta index to confirm causal direction for three-variable paths in a closed system of five variables was investigated and found wanting initially because of imitators. Relationships other than beta-differential comparisons, which made up the beta indices, in the same pairs of regression equations were derived and found to have mathematical consistency for designed causal directions, only two imitators remaining. Causal directions 1-4-5 and 2-4-5, the two designed causal directions were still not eliminated when comparisons of the three correlations among the three path variables, with appropriate control for the effects of other system variables, yielded family-like order of values. A large number of combinations and permutations in the five-variable system still exists unexplored, and obviously micro- and macroscopic studies of same are indicated. It seems possible that with the present model of study, the beta index or at least its elements could find a way into causal theory. Presently, it appears it should be confined to the computer.

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FOOTNOTES

1. Now at Project COD (Title III ESEA, PL 89-10), 52 Ash Street, New Bedford, Mass. 02740.

2. Mathematics for this and other procedures were performed for the most part via programs for IBM 360 at the Boston College Computer Center. The present product was completed under the Title IV program (ESEA, PL 89-10) directed by Dr. John J. Walsh at the College.

3. The model was suggested by Dr. Ronald L. Nuttall, Associate Professor, Department of Psychology, Graduate School of Arts and Sciences, Boston College.

4. Blalock points out that a causal model is characterized by (1) a finite set of explicitly defined variables, (2) assumptions about how the variables are interrelated causally, and (3) assumptions to the effect that outside variables, while operating, do not disturb the causal patterning within the system. The present model meets these conditions; the design further makes the assumptions fact.

5. Numerical order for paths does not refer to direction but merely a name for a given path of variables by numbers assigned. Later, numerical order is established for causal direction in the order given, with numbers hyphenated, to indicate initial-cause, intermediate, and final-effect variables, in their respective or reversed order. For the acausal direction, the reverse order should be obvious from context.

6. R_i are considered exogenous to the system of X_i in the actual causal model, although they are reflected in Equations 3 through 7; they are treated as if they are "random noise." It is also recognized that part correlations instead of partial correlations could have been explored where indicated in Figure 2.

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TABLE 1

Correlation Checks for Model

Prediction Equation	Degrees of Fit[a] Partial Control	Partial Value[c]	Actual Value	Expected Path	Expected Value[d]
$r_{12} = 0$	b	b	0109	b	b
$r_{15} = 0$	$r_{15 \cdot 24}$	0193	4269	$r_{14}r_{45}$	5394
$r_{34} = 0$	$r_{34 \cdot 125}$	-1228	5781	$r_{13}r_{14} + r_{23}r_{24}$	6477
$r_{35} = 0$	$r_{35 \cdot 12}$	0134	5882	$r_{23}r_{25} + r_{23}r_{24}r_{45}$	6217

Note--Values are checked in two ways.
a Decimal points are omitted.
b Check is not necessary by design.
c Value of partial approximates value of prediction equation.
d Value of expected path approximates actual (zero-order) value.
* At df of 200, $r_{01} \geq .181$ (see Arkin and Colton, pp. 24 & 155).

TABLE 2

Beta Studies for Path 145

Step	Comparisons[a] Direction 1-4-5[b] Other[d] X_i	X_1	X_4	$d\beta$	I_β	Direction 5-4-1[b] $d\beta$	X_4	X_5	Other[d] X_i
1		-1822	8720 9847	1127	275	4093	6186 1.0279	-4695	
2A	2 2	0104	6755 6666	0089	262	0340	8514 8274	0355	2 2
2B	3 3	-2706	7989 9093	1104	262	4194	4080 9074	-6250	3 3
3	2, 3 2, 3	0097	6728 6668	0060	444	0135	6159 6024	0200	2, 3 2, 3

Note--Differentials are shown as positive values of differences.

a Decimal points are omitted for values below 1.

b X_5 is predicted.

c X_1 is predicted.

d Other X_i are controlled in regression equations (see Table D).

TABLE 3

Beta Studies for Eight Paths

Other[b] X_i	Comparisons[a] Direction[c]	$d\beta$	I_β	$d\beta$	Direction[c]	Causal Direction[d]
-	2-4-5	1965	202	9721	5-4-2	2-4-5
1		3181	338	9413		2-4-5
3		2161	151	8368		2-4-5
1, 3		2425	382	6352		2-4-5
-	1-3-2	2837	765	2172	2-3-I	2-3-1
4		2722	725	I975		2-3-1
5		2958	371	I098		2-3-I
4, 5		2474	479	1185		2-3-1
-	1-4-2	3172	735	2328	2-4-1	2-4-I
3		3048	681	2079		2-4-1
5		3480	576	2005		2-4-1
3, 5		5064	602	3050		2-4-1
-	2-5-4	1649	427	3861	4-5-2	2-5-4
1		0387	549	0704		2-5-4
3		1634	405	4027		2-5-4
1, 3		0050	563	0088		2-5-4
-	3-1-4	1841	888	2073	4-1-3	3-1-4
2		0995	892	1115		3-1-4
5		0338	462	0727		3-I-4
2, 5		0537	975	0524		4-1-3
-	3-2-4	2165	977	2219	4-2-3	3-2-4
1		0863	908	0951		3-2-4
5		0271	299	0906		3-2-4
1, 5		0507	158	0080		4-2-3
-	3-2-5	1491	462	3228	5-2-3	3-2-5
1		0571	559	1021		3-2-5
4		0019	185	0103		3-2-5
1, 4		0006	600	0010		3-2-5
-	4-2-5	3576	473	7556	5-2-4	4-2-5
1		3484	615	5664		4-2-5
3		2104	372	5662		4-2-5
1, 3		4055	675	6020		4-2-5

Note--Follow Table 2 as model (see Table D for beta coefficients).

a Decimal points are omitted.

b Other X_i are controlled in regression equations.

c Last number of the direction is that of the predicted variable; middle number, that of the intermediate variable from which $d\beta$ is derived.

d Tantative decision is based on I_β.

TABLE 4

Relationships of Variables in Causal Sequence

Causal Direction[b] (xyz)	Type	Correlations[a] c[c]	r_{xz}	c[c]	r_{xy}	c[c]	r_{yz}	Causal[d]
1-4-5	Zero-order		4269		6186		8720	
	Partial	4	2925	2	7223	2	8369	Yes
2-4-5	Zero-order		7288		5293		8720	
	Partial	1	6652	4	6433	12	7221	No
4-2-5	Zero-order		8720		5293		7288	
	Partial	2	8369	1	6652	4	6433	No

Note--See Tables A and E for other correlations.

a Decimal points are omitted.

b Directions are tentative from previous steps; the order of numbers in parentheses is to define correlations listed.

c The number listed is that of the variable controlled.

d Final decision is based on ascending order of correlations by columns from left to right.

* At df of 200, $r_{01} \geq .181$ (see Arkin and Colton, pp. 24 & 155).

TABLE 5

Summary of Mathematical Consistencies

System Number	Control Variables[a]	Comparisons[b] Causal Direction	Acausal Direction	
1	--	$\beta_{cb} - \beta_{cb \cdot a}$	$\beta_{ab} - \beta_{ab \cdot c}$	(23)
		$\frac{r_{ab}}{1 - r_{ab}^2}$	$\frac{r_{bc}}{1 - r_{bc}^2}$	(24)
		$\beta_{ca \cdot b}$	$\beta_{ac \cdot b}$	(25)
		r_{ab}	r_{bc}	(26)
		$r_{ab \cdot c}$	$r_{bc \cdot a}$	(27)
2	d	$\beta_{cb \cdot d} - \beta_{cb \cdot da}$	$\beta_{ab \cdot d} - \beta_{ab \cdot dc}$	(28)
		$\beta_{ca \cdot bd}$	$\beta_{ac \cdot bd}$	(29)
		$r_{ab \cdot d}$	$r_{cb \cdot d}$	(30)
		$r_{ab \cdot cd}$	$r_{cb \cdot ad}$	(31)
3	e	$\beta_{cb \cdot e} - \beta_{cb \cdot ea}$	$\beta_{ab \cdot e} - \beta_{ab \cdot ec}$	(32)
		$\beta_{ca \cdot be}$	$\beta_{ac \cdot be}$	(33)
		$r_{ab \cdot e}$	$r_{cb \cdot e}$	(34)
		$r_{ab \cdot ce}$	$r_{cb \cdot ae}$	(35)
4	d, e	$\beta_{cb \cdot de} - \beta_{cb \cdot dea}$	$\beta_{ab \cdot de} - \beta_{ab \cdot dec}$	(36)
		$\beta_{ca \cdot bde}$	$\beta_{ac \cdot bde}$	(37)
		$r_{ab \cdot de}$	$r_{cb \cdot de}$	(38)
		$r_{ab \cdot cde}$	$r_{cb \cdot ade}$	(39)

Note--Generalizations were found for five-variable system and subsystems.

a This includes other than the ones included in three-variable paths.

b Causal direction, a-b-c, value is less than acausal direction, c-b-a, value for path abc with other system variables, d and e, introduced where appropriate; values are grouped vertically as comparisons of beta differentials on intermediate variables, beta coefficients of end variables, and other path correlations, respectively. Absolute-value sysmbols are omitted.

TABLE 6

Variance Accounted For by X_i and R_i

Predicted Variance[b] (m•wxyz)	Variance[a] V_{mw}	V_{mx}	V_{my}	V_{mz}	V_{total}
3•12) 3•67)[c]	3566	2694	-	-	6260
3•128) 3•678)[c]	3794	3060	3146	-	1.0000
4•12) 4•67)[c]	3793	2766	-	-	6558
4•129) 4•679)[c]	3835	3042	3123	-	1.0000
5•24	2705	5891	-	-	8596
5•2410	2706	5890	1404	-	1.0000
5•679	1810	5535	1252	-	8597
5•67910	2257	4499	1840	1404	1.0000

Note--R_i, where i is 1 through 5 in the original design, are set as variables 6 through 10, respectively, to facilitate their introduction into the system; variances are computed after Baggaley (p. 55), where total variance equals the sum of products of beta coefficient and respective correlation for each term (predictor-predicted) in the regression system.

a Decimal points are omitted for values below 1.

b The order of numbers in parentheses is to define components listed.

c X_1 and X_2 are the same as R_1 and R_2, respectively; hence, variables 6 and 7, respectively.

TABLE A

Correlation Matrix of X_i and R_i[a]

Variable		X_i				R_i				
		2	3	4	5	1	2	3	4	5
X	1	0109	6001	6186	4269	1.0000	0109	-0624	-0131	0190
	2		5222	5293	7288	0109	1.0000	-1147	-0885	-0346
	3			5781	5882	6001	5222	5101	-1597	-0047
	4				8720	6186	5293	-1997	5301	-0112
	5					4269	7288	-1763	3200	3542
R	1						0109	-0624	-0131	0190
	2							-1147	-0885	-0346
	3								-1614	0057
	4									0053

Note--X_i are endogenous variables; R_i, exogenous; that is, internal and external, respectively, to the system.

a Decimal points are omitted for values below 1.

* At df of 200, $r_{01} \geq .181$ (see Arkin and Colton, pp. 24 & 155).

TABLE B

Check of Means in Equations

Equation	Left Member	Right Member
3	496.5521	496.5522
4	498.6933	498.6934
5	1478.4710	496.5521 + 498.6933 + 483.2257 = 1478.4681
6	1479.1044	496.5521 + 498.6933 + 483.8592 = 1479.1046
7	2476.2700	498.6933 + 1479.1044 + 498.4725 = 2476.2702

TABLE C

Check of Means with Regression Equations

Equation	Left Member	Right Member[a]
5	1478.4710	.9404(496.5521) + .8800(498.6933) + 572.6838
		466.9576 + 438.8501 + 572.6838 = 1478.4915
6	1479.1044	.9885(496.5521) + .9095(498.6933) + 534.7444
		490.8418 + 453.5616 + 534.7444 = 1479.1478
7	2476.2700	.9614(498.6933) + 1.0055(1479.1044) + 509.6130
		479.4436 + 1487.2395 + 509.6130 = 2476.2961

a Equations are in the form of regression equations, each term corresponding to the term of like variable in Equations 5, 6, and 7.

TABLE D

Beta Coefficients of Predictor X_i[a]

Predicted X_i	Predictor X_i									
	C^b	X_1	C^b	X_2	C^b	X_3	C^b	X_4	C^b	X_5
1	-	-	-	0109	-	6001	-	6186	-	4269
			3	-4159	2	8173	2	8514	2	8934
			4	-4398	4	3643	3	4080	3	1129
			5	-6402	5	5337	5	1.0279	4	-4695
			34	-6085	24	5618	23	6159	23	6084
			35	-7685	25	6435	25	8274	24	0355
			45	-4530	45	4432	35	9074	34	-6250
			345	-6402	245	5617	235	6024	234	0200
2	-	0109	-	-	-	5222	-	5293	-	7288
	3	-4727			1	8059	1	8465	1	8855
	4	-5127			4	3246	3	3417	3	6447
	5	-3671			5	1430	5	-4428	4	1.1149
	34	-7472			14	5968	13	6465	13	7073
	35	-5544			15	4388	15	-0948	14	9559
	45	-3385			45	1923	35	-4951	34	1.0474
	345	-5582			145	4397	135	0113	134	6985
3	-	6001	-	5222	-	-	-	5781	-	5882
	2	5945	1	5157			1	3351	1	4060
	4	3928	4	3003			2	4192	2	4429
	5	4268	5	1994			5	2721	4	3510
	24	7060	14	6108			12	-1819	12	-1410
	25	6536	15	6178			15	-2414	14	5855
	45	4995	45	2900			25	4005	24	0276
	245	7060	145	6098			125	-1836	124	0025
4	-	6186	-	5293	-	5781	-	-	-	8720
	2	6129	1	5227	1	3234			1	7434
	3	4245	3	3128	2	4148			2	1.0369
	5	3012	5	-2263	5	0997			3	8133
	23	7124	13	6090	12	-1674			12	7821
	25	2852	15	-0437	15	-0791			13	7755
	35	3350	35	-2534	25	1359			23	9767
	235	3389	135	0070	125	-0822			123	7705
5	-	4269	-	7288	-	5882	-	8720	-	-
	2	4190	1	7242	1	5189	1	9847		
	3	1154	3	5797	2	2855	2	6755		
	4	-1822	4	3712	4	1263	3	7989		
	23	4848	13	7813	12	-1107	12	6666		
	24	0104	14	3758	14	2249	13	9093		
	34	-2706	34	3693	24	0065	23	6728		
	234	0097	134	3758	124	0010	123	6668		

Note--This table is the basis for Tables 2 and 3.
a Decimal points are omitted for values below 1.
b The numbers are those of the variables controlled.

TABLE E

Partial Correlation Matrix of X_i[a]

Variable X_i	C[b]	X_2	C[b]	X_3	C[b]	X_4	C[b]	X_5
1	-	0109	-	6001	-	6186	-	4269
	3	-4434	2	6970	2	7223	2	6188
	4	-4749	4	3783	3	4162	3	1142
	5	-4848	5	3374	5	5565	4	-2925
	34	-6743	24	6298	23	6624	23	5431
	35	-6527	25	6486	25	4857	24	0193
	45	-3916	45	4705	35	5513	34	-4113
	345	-5863	245	6297	235	4518	234	0140
2			-	5222	-	5293	-	7283
			1	6447	1	6652	1	8008
			4	3122	3	3269	3	6113
			5	1688	5	-3166	4	6433
			14	6037	13	6275	13	7434
			15	5207	15	-0644	14	5994
			45	2362	35	-3542	34	6219
			145	5178	135	0089	134	5119
3					-	5781	-	5882
					2	4170	2	3556
					5	1647	4	2106
					1	3292	1	4590
					12	-1745	12	-1249
					15	-1382	14	3629
					25	2333	24	0134
					125	-1228	124	0016
4							-	8720
							1	8556
							2	8369
							3	8061
							12	7221
							13	8398
							23	8106
							123	7168

a Decimal points are omitted.
b The numbers are those of the variables controlled.
* At df of 200, $r_{01} \geq .181$ (see Arkin and Colton, pp. 24 & 155).

TABLE F

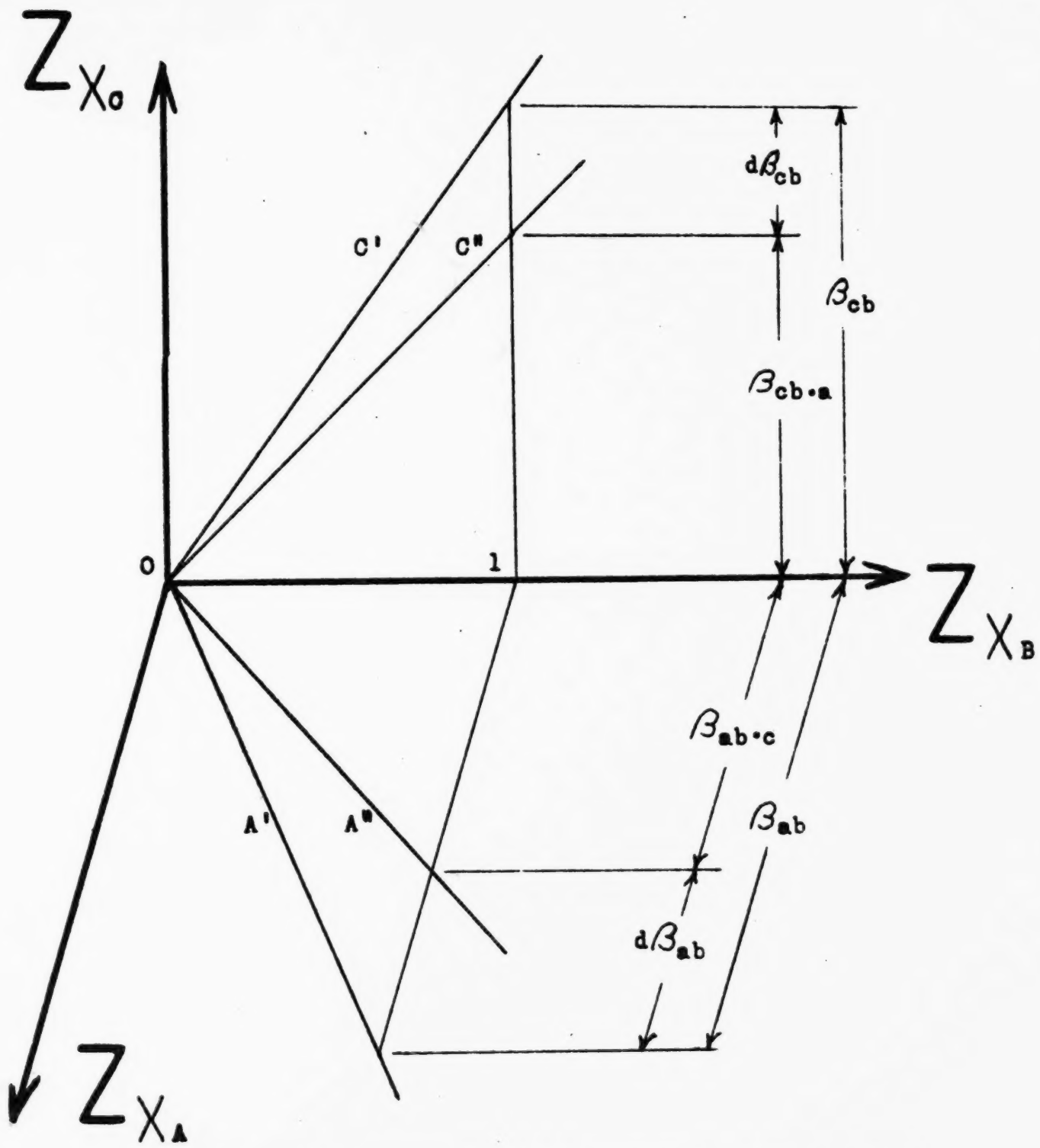
Mathematical Consistencies for Causal Directions

Comparisons[a]	Selected Causal Directions[b] 1-4-5	2-4-5	2-3-1	2-4-1	2-5-4	3-1-4	3-2-4	3-2-5	4-2-5
23	1	1	1	1	1	1	1	1	1
24	1	1	1	1	1	1	1	1	1
25	1	1	1	1	1	1	1	1	1
26	1	1	1	1	1	1	1	1	1
27	1	1	1	1	1	1	1	1	1
28	1	1	1	1	1	1	1	1	1
29	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1
31	1	1	1	1	1	1	1	1	1
32	1	1	1	1	1	1	1	1	1
33	1	1	0	0	1	1	1	1	1
34	1	1	1	1	1	1	1	1	1
35	1	1	1	1	1	1	1	1	1
36	1	1	1	1	1	0	0	1	1
37	1	1	0	0	1	1	1	1	1
38	1	1	1	1	1	0	0	0	1
39	1	1	0	1	1	0	0	0	1

Note--For comparisons of actual values, see Tables 2, 3, A, D, and E.

a Comparisons are those of Inequalities 23 through 39, as generalized in Table 5, in the same order.

b Causal directions were selected as a result of comparisons of first sets of beta differentials; that is, Inequality 23. Then 1 means YES that the comparison of actual values substituted makes the inequality true; 0, NO. Since another step is needed to eliminate two directions, 2-5-4 and 4-2-5, Table 4 is referred to.



Assume X_a causes X_b which causes X_c, in Path ABC.

CAUSAL DIRECTION: A-B-C

$$z'_{X_c} = \beta_{cb} z_{X_b}$$

$$z''_{X_c} = \beta_{cb \cdot a} z_{X_b} + \beta_{ca \cdot b} z_{X_a}$$

$$d\beta_{cb} = \overline{|\beta_{cb} - \beta_{cb \cdot a}|}$$

ACAUSAL DIRECTION: C-B-A

$$z'_{X_a} = \beta_{ab} z_{X_b}$$

$$z''_{X_a} = \beta_{ab \cdot c} z_{X_b} + \beta_{ac \cdot b} z_{X_c}$$

$$d\beta_{ab} = \overline{|\beta_{ab} - \beta_{ab \cdot c}|}$$

$$d\beta_{cb} < d\beta_{ab}$$

$$I_\beta = d\beta_{cb} / d\beta_{ab} \quad \text{where } 0 < I_\beta < 1$$

FIGURE 1

THE BETA CONSTANCY PRINCIPLE AS BASIS FOR THE BETA INDEX

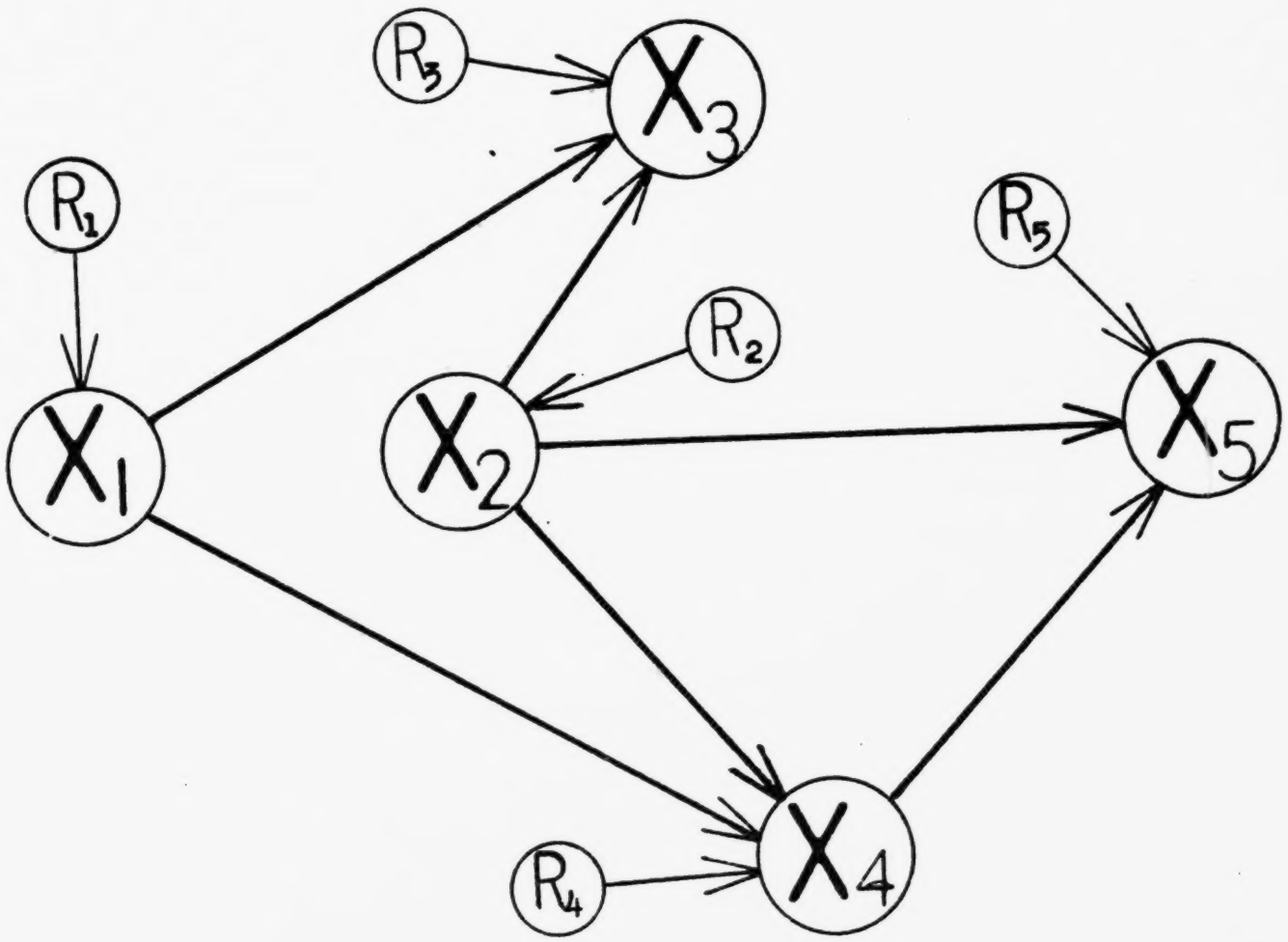


FIGURE 2

CAUSAL MODEL

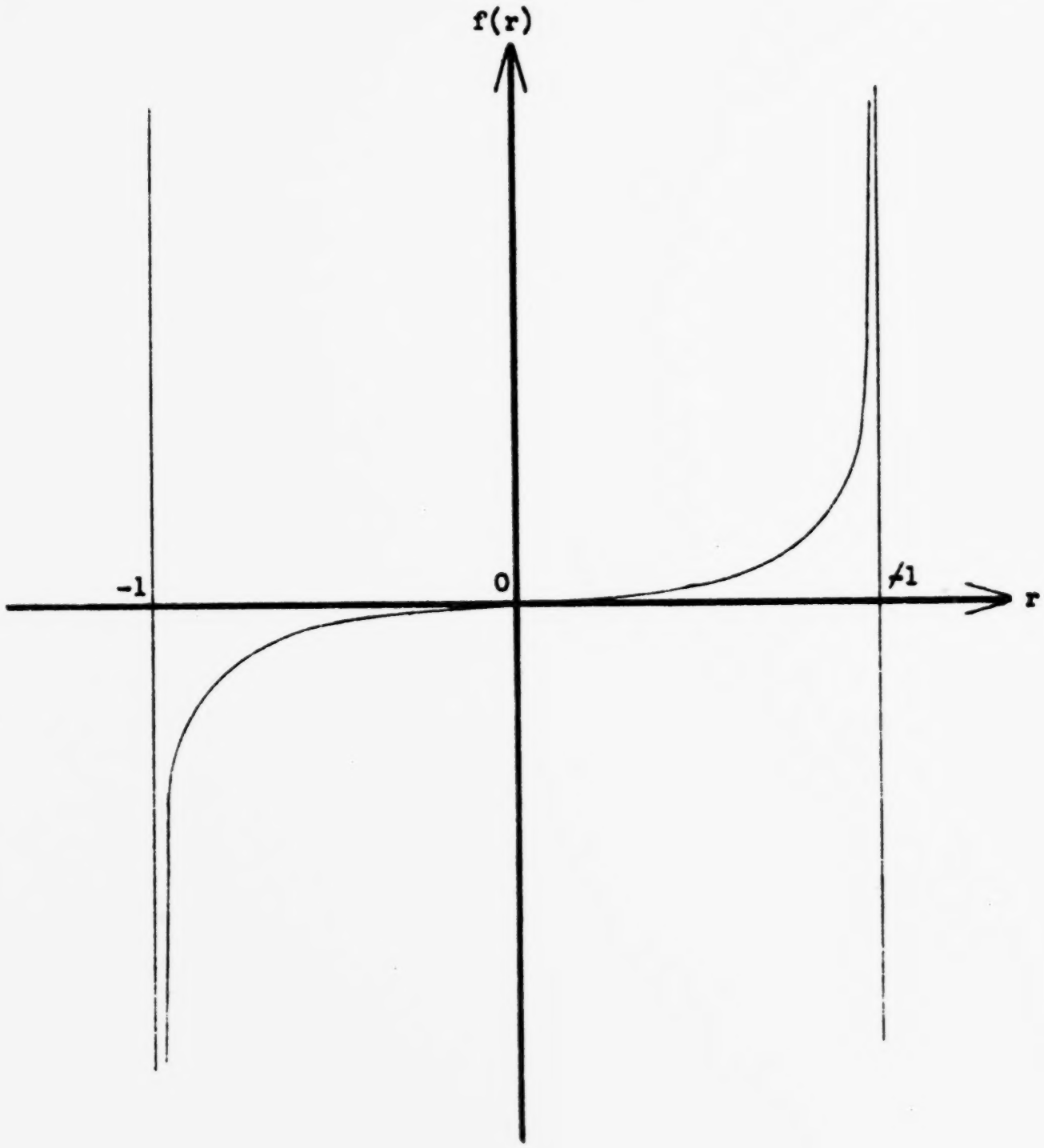


FIGURE A

FUNCTIONAL RELATIONSHIPS OF A BETA-DIFFERENTIAL ELEMENT

PART II

READING MANUSCRIPT

Part II is for verbatim reading. It is a condensation of Part I, which is the report of the study itself, such that it may be read in 12-15 minutes. In order to clarify main points while also maintaining brevity, four transparencies are resorted to, with overlays as necessary, for use with the overhead projector. Microphone and loudspeaker will be available for an expected attendance of 150 in the New York Room of the Statler Hilton Hotel, 1:20-3:40 p.m., Friday, February 7, 1969. No assistance will be necessary, nor substitute reader. Dr. Jerome E. Doppelt of The Psychological Corporation is chairman.

A BETA INDEX TO CONFIRM CAUSAL DIRECTIONS IN A CLOSED SYSTEM OF FIVE VARIABLES

READING MANUSCRIPT

GEORGE A. NIGRO

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GOOD AFTERNOON, LADIES AND GENTLEMEN.

THE RESEARCHER OFTEN HAS DIFFICULTY CONVINCING HIMSELF AND OTHERS ABOUT HIS FINDINGS ON CAUSAL PATHS WITHIN A SYSTEM OF VARIABLES, REGARDLESS OF RATIONAL CONCLUSIONS, FOR HE MAY NOT HAVE INCLUDED ALL RELEVANT VARIABLES. BLALOCK, IN PULLING TOGETHER MATERIALS ON CAUSAL MODELS, ATTEMPTED TO SPELL OUT THE RULES OF CAUSAL INFERENCE. HIS MAIN STRATEGY WAS TO STUDY BETA COEFFICIENTS IN REGRESSION EQUATIONS; HIS STRATEGY SERVED AS THE SPRINGBOARD FOR THE PRESENT STUDY.

BRIEFLY, A CAUSAL MODEL INCLUDES A FINITE SET OF EXPLICITLY DEFINED VARIABLES, ASSUMED TO BE INTERRELATED CAUSALLY, ALONG LOGICALLY ASCERTAINED PATHS. IT IS ALSO ASSUMED THAT OUTSIDE VARIABLES, WHILE OPERATING, DO NOT DISTURB THE CAUSAL PATTERNING WITHIN THE SYSTEM. THE VARIABLES IN THE SYSTEM HAVING CAUSAL RELATIONSHIPS ARE NOT MERELY ASSOCIATED OR JOINED IN SEQUENCE. BUT IF X PRODUCES Y, THEN ALSO A CHANGE IN X PRODUCES A CHANGE IN Y. IF THE MODEL IS FOUND INADEQUATE, THEN ADDITIONAL VARIABLES MUST BE INTRODUCED, WITHIN PRACTICAL LIMITS. ONCE FOUND ADEQUATE, THE MODEL MAY THEN BE USED FOR OTHER DESIGNS, WHICH CAN BE TRANSLATED INTO TESTABLE HYPOTHESES.

IN THE EXPLORATORY CONSTRUCTION OF A BLALOCKIAN CAUSAL MODEL, IT WAS DISCOVERED THAT THE BETA COEFFICIENT OF AN INTERMEDIATE VARIABLE, IN A CAUSAL DIRECTION, REMAINS RELATIVELY CONSTANT, AS OTHER VARIABLES ARE INTRODUCED AND CONTROLLED IN STEPPED REGRESSION; WHEREAS THAT IN THE ACAUSAL DIRECTION CHANGES NOTICEABLY--THE BETA CONSTANCY PRINCIPLE, UPON WHICH A BETA INDEX WAS DEVELOPED, AS ILLUSTRATED IN FIGURE 1.

(TRANSPARENCY 1—1 ON.)

ASSUME THAT A→B→C.

HERE WE HAVE TWO STANDARD-SCORE AXES FOR VARIABLES B AND C. LET C' REPRESENT THE LINE FOR PREDICTING C FROM B. LET C" REPRESENT THE LINE FOR PREDICTING C FROM B, BUT WITH A THIRD VARIABLE A STEPPED IN AND CONTROLLED.

(TRANSPARENCY 1—2 OVERLAY.)

HERE WE HAVE ADDED A THIRD STANDARD-SCORE AXIS FOR VARIABLE A. LET A' REPRESENT THE LINE FROM WHICH A IS PREDICTED FROM B. LET A" REPRESENT THE LINE FOR PREDICTING A FROM B, BUT WITH THE THIRD VARIABLE C STEPPED IN AND CONTROLLED.

(TRANSPARENCY 1—3 OVERLAY.)

LET US THEN DROP A PERPENDICULAR FROM EACH OF THE FOUR PREDICTION LINES TO A DISTANCE OF 1 UNIT FROM THE ORIGIN, ALONG THE AXIS FOR B MEASUREMENTS.

(TRANSPARENCY 1—4 OVERLAY.)

THEN THE PERPENDICULUR DISTANCES TO EACH OF THE FOUR LINES REPRESENT THE SLOPES, OR BETA COEFFICIENTS, OF THE PREDICTOR VARIABLE B, IN PREDICTING C AND A. THE DIFFERENCE BETWEEN THE SLOPES IN A GIVEN DIRECTION THEN BECOMES CLEAR.

LET US CALL THE DIFFERENCE IN THE CAUSAL DIRECTION THE BETA DIFFERENTIAL ON THE INTERMEDIATE VARIABLE B, FOR PREDICTING THE FINAL-EFFECT VARIABLE C, WHILE CONTROLLING FOR THE INITIAL-CAUSE VARIABLE A STEPPED INTO REGRESSION; THIS HAS THE SYMBOL OF $d\beta_{cb}$.

LET US CALL THE DIFFERENCE IN THE ACAUSAL DIRECTION THE BETA DIFFERENTIAL ON THE INTERMEDIATE VARIABLE B, FOR PREDICTING THE INITIAL-CAUSE VARIABLE A, WHILE CONTROLLING FOR THE FINAL-EFFECT VARIABLE C STEPPED INTO REGRESSION; THIS HAS THE SYMBOL OF $d\beta_{ab}$.

IT SHOULD BE OBVIOUS THAT THE DIFFERENTIAL IN THE CAUSAL DIRECTION IS LESS THAN THAT IN THE ACAUSAL DIRECTION, AS SHOWN IN INEQUALITY 1.

FOR CLEARCUT 3-VARIABLE CAUSAL PATHS, THE INEQUALITY BETWEEN THE MAGNITUDES OF THE TWO BETA DIFFERENTIALS PERSISTED, AS

REMAINING VARIABLES WERE STEPPED IN AND CONTROLLED, APPARENTLY A CASE OF ASSYMETRY EMPHASIZED BY BLALOCK.

THE TWO MEMBERS OF THE INEQUALITY WERE THEN CONVERTED TO THE FORM OF A RATIO, CALLED THE BETA INDEX. THE SMALLER OF THE TWO DIFFERENTIALS, THAT FOR THE CAUSAL DIRECTION, BECAME THE NUMERATOR. THE LARGER, THAT IN THE ACAUSAL DIRECTION, BECAME THE DENOMINATOR. HENCE, THE BETA INDEX IS THE RATIO OF THE DIFFERENTIAL IN THE CAUSAL DIRECTION TO THAT IN THE ACAUSAL DIRECTION, OR EQUATION 2.

THE VALUE OF THIS INDEX, MOREOVER, WAS MADE TO VARY BETWEEN 0 AND +1; THE CLOSER TO 0, THE STRONGER THE CAUSAL DIRECTION.

(TRANSPARENCY 1 OFF.)

THE PROBLEM THEN EVOLVED AS TO WHETHER THE BETA INDEX COULD BE DEMONSTRATED IN A CLOSED SYSTEM OF VARIABLES, DERIVED FROM DUMMY DATA AND RELATED BY KNOWN CAUSAL PATHS, BY A SYSTEM OF EQUATIONS.

METHOD

200 x 5 NORMALIZED RANDOM NUMBERS, R_i, WERE GENERATED AND SUBSTITUTED INTO FIVE EQUATIONS, EQUATIONS 3—7 IN YOUR PAPER, TO OBTAIN 5 SCORES, X_i, FOR EACH OF 200 PSEUDO-PEOPLE. THESE EQUATIONS FORM THE MODEL PORTRAYED IN FIGURE 2.

(TRANSPARENCY 2—1 ON.)

IT SHOULD BE REMEMBERED THAT EXTERNAL VARIABLES R_i ARE EVER PRESENT, ALTHOUGH OPERATING NOT TO DISTURB THE CAUSAL PATTERNING WITHIN THE SYSTEM.

(TRANSPARENCY 2—2 OVERLAY.)

(TRANSPARENCY 2 OFF.)

THE b COEFFICIENTS IN THE 5 EQUATIONS WERE SET EQUAL TO 1 TO FIND X_3, X_4, AND X_5, IN ORDER TO SIMPLIFY MANIPULATION AND ANALYSIS FOR THE PURPOSE OF THE PRESENT STUDY. X_i THEN SERVED AS INPUT IN STEPPED MULTIPLE-REGRESSION ANALYSIS, THE OUTPUT CONSISTING OF THE REQUIRED BETA COEFFICIENTS AND OTHER ANCILLARY DATA. SEVERAL CHECKS WERE RUN ON THE OUTPUT BEFORE PROCEEDING WITH THE STUDY. THESE CHECKS ARE FULLY DETAILED ON PAGE 2 AND PERTINENT TABLES IN YOUR PAPER.

THE BASIC PROCEDURE FOR BETA-DIFFERENTIAL COMPARISONS INCLUDED 4 STEPS AND IS BEST ILLUSTRATED BY TABLE 2.

(TRANSPARENCY 3 ON.)

IN TABLE 2, DATA FOR PATH 145 ARE STUDIED. NOTE THAT THE LEFT SIDE OF THE TABLE INCLUDES BETA COEFFICIENTS FOR THE DESIGNED

CAUSAL DIRECTION; THAT IS, IN PREDICTING X_5. WHEREAS, THE RIGHT SIDE OF THE TABLE INCLUDES THOSE FOR THE ACAUSAL DIRECTION; THAT IS, IN PREDICTING X_1.

LOOK ACROSS THE TOP ROW OF STEP 1. NOTE THAT β_{54} IS GREATER THAN β_{14}--THEY TELL US NOTHING AS TO CAUSAL DIRECTION. IN THE SECOND ROW, THE THIRD, OR END, VARIABLE IS STEPPED IN FOR EACH DIRECTION, TO PREDICT X_5 AND X_1. NOTE THE REVERSAL; THE NEW $\beta_{14\cdot5}$ IS GREATER THAN $\beta_{54\cdot1}$, BUT THEY STILL TELL US NOTHING AS TO CAUSAL DIRECTION.

BUT THERE ARE OBVIOUSLY CHANGES IN EACH DIRECTION. THESE CHANGES IN BETA COEFFICIENTS ARE THE BETA DIFFERENTIALS; $d\beta_{54}$ IS OBVIOUSLY LESS THAN $d\beta_{14}$. THE CHANGE IN THE DIRECTION OF PREDICTING X_5, THE DESIGNED FINAL-EFFECT VARIABLE, IS LESS THAN IN THE OPPOSITE DIRECTION OF PREDICTING X_1, THE DESIGNED INITIAL-CAUSE VARIABLE. THE TWO VALUES GIVE US A BETA INDEX OF .275, A VALUE CLOSE TO 0. TENTATIVELY THEN, BASED ON THE CONCEPT ALREADY DESCRIBED, DIRECTION 1→4→5 IS CAUSAL.

(MOVE DOWN 1 STEP.)

IN LIKE MANNER, IN STEP 2A, A SYSTEM THAT INCLUDES A FOURTH VARIABLE, X_2, IS CONSIDERED. IN THE TOP ROW, THE FIRST REGRESSION EQUATION INCLUDES X_5 PREDICTED FROM X_4 AND X_2 FIRST, AND THEN WITH X_1 STEPPED IN, ON THE SECOND ROW. X_1, ON THE RIGHT, IS PREDICTED FROM THE SAME X_4 AND X_2, AND THEN WITH X_5 STEPPED IN. NOTE THAT X_2 IS A SYSTEM VARIABLE, BUT IT IS NOT LISTED, SINCE IT IS NOT

STUDIED; ITS BETA COEFFICIENT MAY BE FOUND IN TABLE D. AGAIN THE BETA DIFFERENTIAL IS LESS IN PREDICTING X_5 THAN X_1, AGAIN FOR A LOW BETA INDEX.

(MOVE DOWN 1 STEP.)

STEP 2B REPEATS THE PROCEDURE FOR STEP 2A, BUT WITH SYSTEM VARIABLE X_3 REPLACING X_2.

(MOVE DOWN TO LAST STEP.)

LASTLY, STEP 3, THE SYSTEM FOR ALL 5 VARIABLES, INCLUDES BOTH X_2 AND X_3, BEFORE THE END VARIABLE IS STEPPED IN. IN EACH SYSTEM OF INCREASING NUMBERS OF VARIABLES, THE BETA INDEX IS CLOSE TO 0. THE BETA DIFFERENTIAL IN PREDICTING X_5, THE DESIGNED FINAL-EFFECT VARIABLE, IS LESS THAN THAT IN PREDICTING X_1, THE DESIGNED INITIAL-CAUSE VARIABLE. BASED ON THE BETA-INDEX CONCEPT, DIRECTION 1→4→5 PERSISTED AS CAUSAL.

(TRANSPARENCY 3 OFF.)

(TRANSPARENCY 2—1 ON AGAIN.)

PATH 245 RESULTED IN SIMILAR BETA-DIFFERENTIAL PATTERNS. IN OTHER WORDS, WHAT WAS EXPECTED FOR THE TWO CLEARCUT CAUSAL DIRECTIONS, 1→4→5 AND 2→4→5 IN THE DESIGN, WAS FOUND.

BUT IMITATORS OF THESE DESIGNED CAUSAL DIRECTIONS WERE ALSO FOUND, THUS WEAKENING THE UTILITY OF THE BETA INDEX.

EXPLORATORY STUDY OF REGRESSION EQUATIONS REVEALED SOME AMPLIFYING INFORMATION ABOUT DIRECTIONS 1—4—5 AND 2—4—5.

(TRANSPARENCY 2 OFF.)

THE BETA COEFFICIENTS FOR END VARIABLES COMPARED THE SAME AS THE BETA DIFFERENTIALS IN LIKE DIRECTIONS.

(TRANSPARENCY 3 ON AGAIN.)

NOTE THAT THE BETA COEFFICIENT OF X_1, IN PREDICTING X_5, IS LESS THAN THAT OF X_5 IN PREDICTING X_1, AT EACH STEP IN THE TABLE. BUT DIRECTIONS 2—5—4 AND 4—2—5 STILL HELD UP AS IMITATORS OF 1—4—5 AND 2—4—5. UTILITY OF THE BETA INDEX REMAINED IN QUESTION.

(TRANSPARENCY 3 OFF.)

BLALOCK OBSERVED THAT THE CORRELATION BETWEEN VARIABLES FARTHEST APART IN CAUSAL SEQUENCE SHOULD BE SMALLEST. IT WAS ALSO REASONED THAT THE MOST RECENT RELATIONSHIP IN THE CAUSAL SEQUENCE SHOULD BE GREATER IN MAGNITUDE THAN ANY CAUSALLY PRIOR RELATIONSHIP. TOGETHER, THESE TWO POINTS SEEMED TO BE SIMILAR TO THE FAMILY-LIKE

RELATIONSHIP AMONG GRANDFATHER, FATHER, AND SON OVER TIME. SUCH RELATIONSHIPS, WITH APPROPRIATE CONTROLS FOR THE EFFECTS OF OTHER SYSTEM VARIABLES, WERE FOUND ONLY FOR DIRECTIONS 1--4—5 AND 2—4—5. THIS FINDING SEEMS TO EMANATE MIXED FEELINGS ABOUT THE BETA INDEX. BY A SCREENING PROCEDURE, THE TWO DESIGNED DIRECTIONS IN CAUSAL SEQUENCE OF 3-VARIABLE PATHS HELD UP, YET ONE WONDERS IF CORRELATIONS ALONE COULD BE STUDIED TO CONFIRM CAUSAL DIRECTIONS. BUT THE ONE FORMIDABLE OBSTACLE IN SUCH STUDY WOULD BE TO PLAY CHESS ON A CORRELATION MATRIX INCLUDING ALL PARTIALS.

THE MATHEMATICS WERE THEN STUDIED TO FIND OUT WHY THE BETA INDEX AND END-VARIABLE BETA COEFFICIENTS WERE PERSISTENTLY PATTERNED FOR CAUSAL DIRECTIONS.

DISCUSSION

THE MATH IS TREATED IN DETAIL IN THE DISCUSSION SECTION OF YOUR PAPER. NOTE INEQUALITY 8 FOR BETA DIFFERENTIALS AND INEQUALITIES 11, 13, 15, AND 17 FOR END-VARIABLE BETA COEFFICIENTS. BUT MOST REVEALING, AND SUPPORTIVE TO THE IDEA OF CORRELATIONAL STUDY, ARE INEQUALITIES 10, 12, 14, 16, AND 18, DERIVED FROM CORRESPONDING BETA COMPARISIONS, AND INEQUALITIES 19—22, ARRIVED AT BY LOGIC. THESE COMPARISONS ARE SUMMARIZED IN TABLE 5 FOR SYSTEMS OF 3, 4, AND 5 VARIABLES; THEY ARE ALSO GENERALIZED HERE.

(TRANSPARENCY 4 ON.)

ASSUME CAUSAL DIRECTION A—B—C IN A SYSTEM. THESE INEQUALITIES WOULD SEEM TO HOLD TRUE, WHERE K MAY BE ZERO OR MORE OTHER RELEVANT VARIABLES, CONTROLLED IN THE SYSTEM. ALSO NOTE THE FAMILY-LIKE COMPARISONS OF CORRELATIONS FOR DIRECTION A—B—C.

(TRANSPARENCY 4 OFF.)

CONCLUSIONS

THE PROBLEM OF USING A BETA INDEX TO CONFIRM CAUSAL DIRECTION WAS INVESTIGATED AND FOUND WANTING INITIALLY BECAUSE OF IMITATORS.

OTHER RELATIONSHIPS WERE FOUND THAT DISPLAYED MATHEMATICAL CONSISTENCY FOR DESIGNED CAUSAL DIRECTIONS. THESE POINTED IN THE DIRECTION OF CORRELATIONAL RATHER THAN BETA STUDY.

ALTHOUGH THE BETA-INDEX CONCEPT IS STRONGLY SUPPORTED, ITS USAGE MIGHT WELL BE CONFINED TO COMPUTERS PRESENTLY.

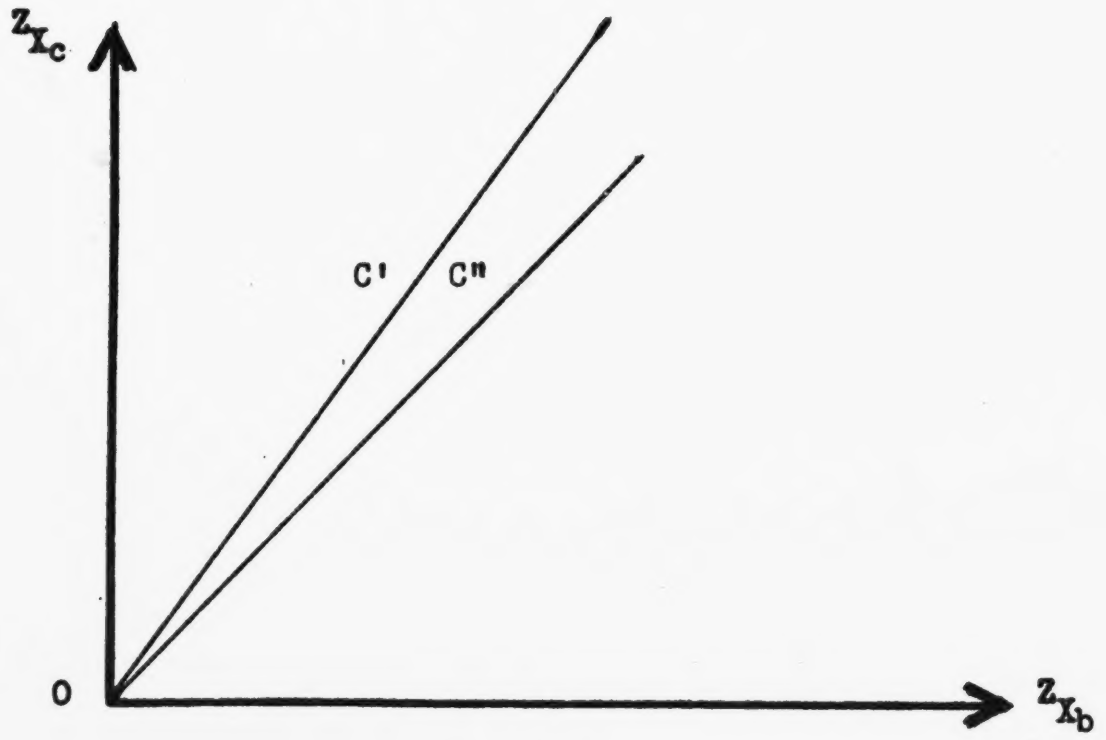
THE ELEMENTS OF THE INDEX, HOWEVER, THE INEQUALITIES, SHOULD BE CONSIDERED IN THE STUDY OF OTHER CAUSAL MODELS.

THE REAL ADVANTAGE OF THE BETA INDEX OVER CORRELATIONAL STUDY SEEMS TO BE IN TAKING OUT THE CHESSBOARD FRUSTRATION OF INSPECTING A CORRELATION MATRIX THAT INCLUDES ALL PARTIALS. IN ANY EVENT, THE STUDY SEEMS TO HAVE BEEN WORTHWHILE AND PERSONALLY INTERESTING.

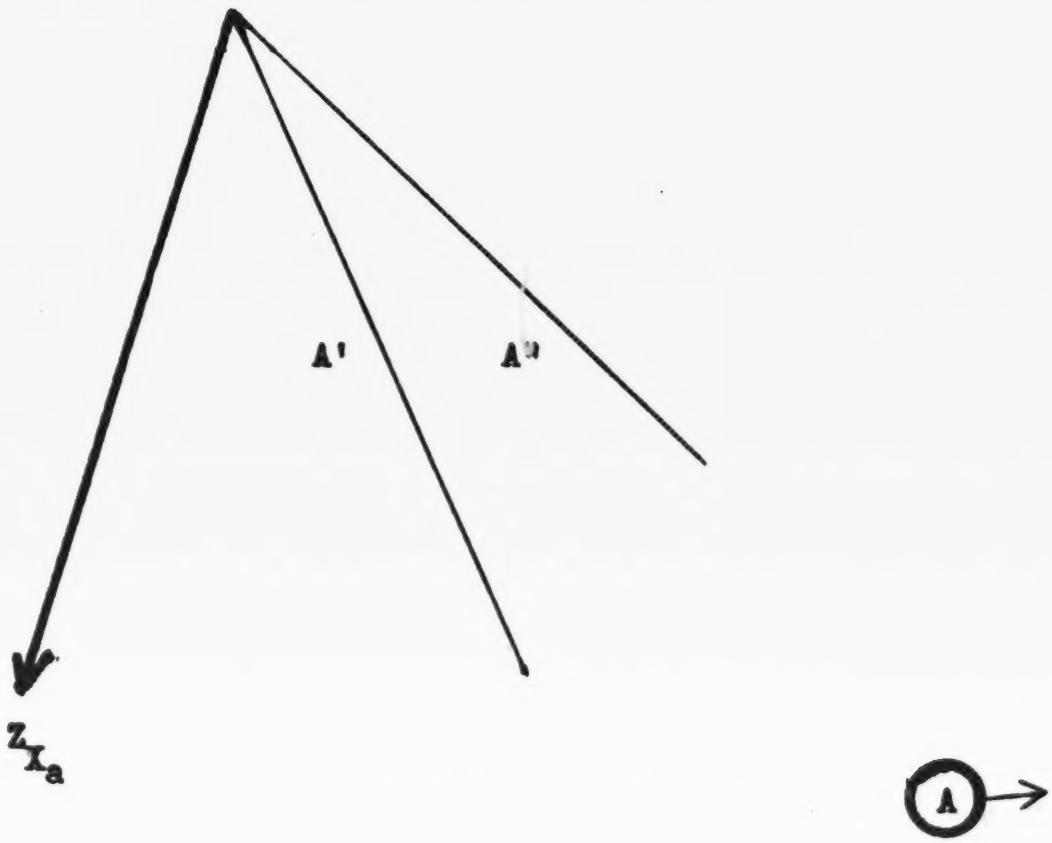
IN LEADING THE CAUTIOUS, PATIENT LISTENER AROUND ROBIN HOOD'S BARN, THE AUTHOR HOPES THAT SOME BENEFIT HAS BEEN DERIVED FROM THE

PRESENTATION. AS A PROFESSOR ONCE SAID AT THE END OF HIS LONG LECTURE, "THE LONGER THE SPOKE, THE GREATER THE TIRE."

THANK YOU.

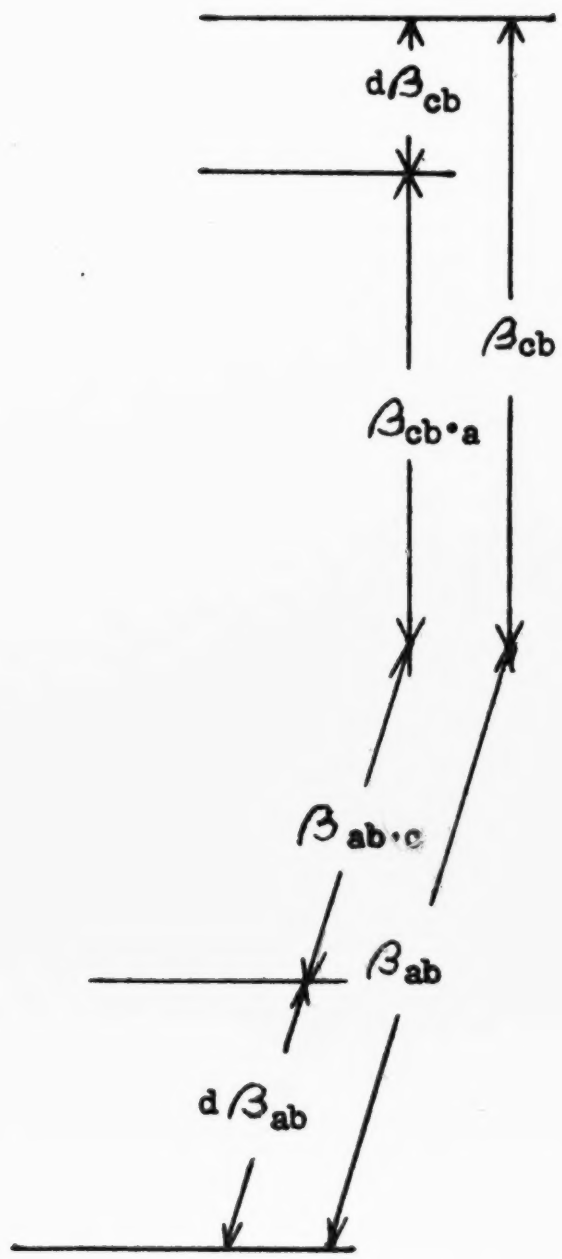


TRANSPARENCY 1-1

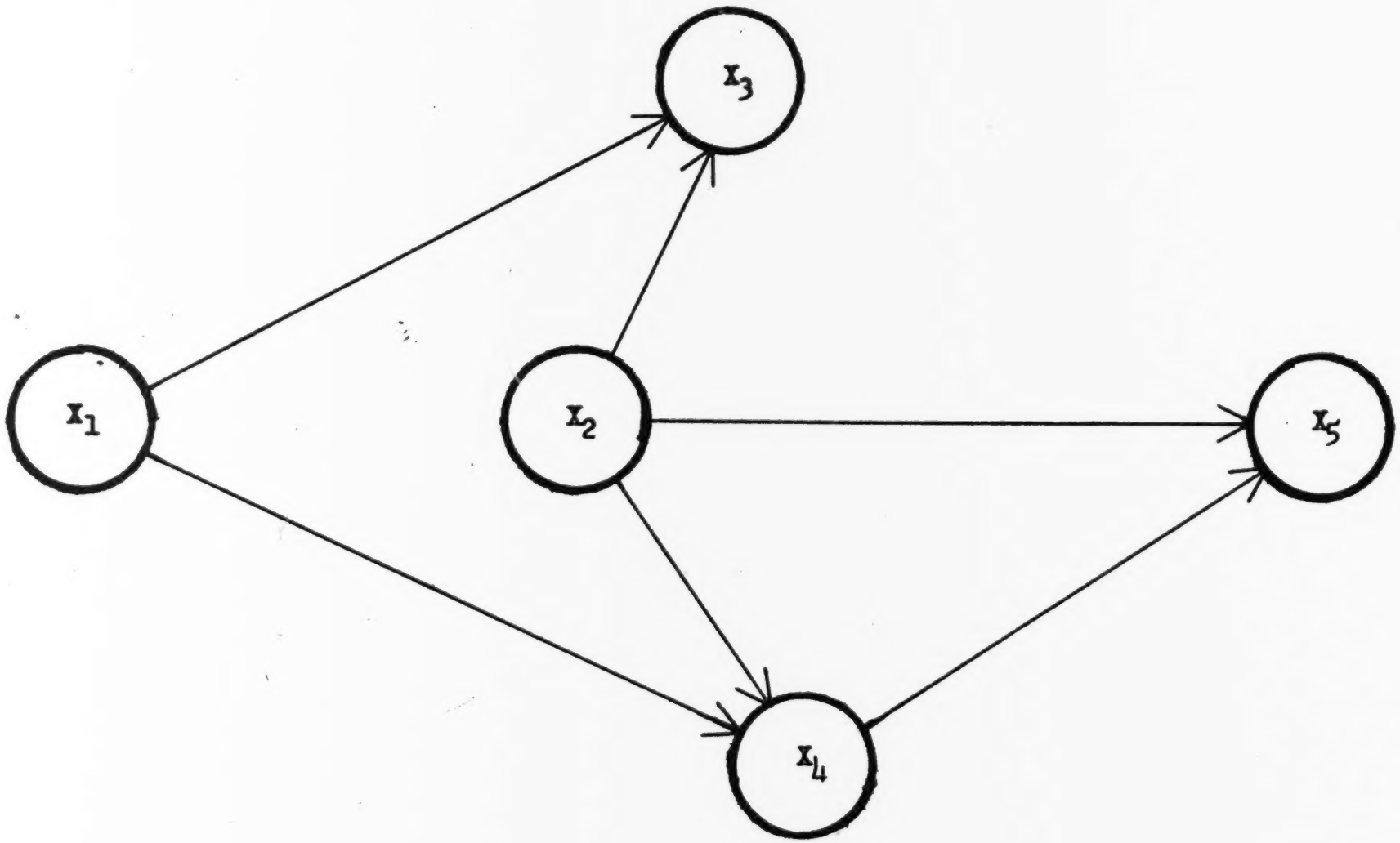


TRANSPARENCY 1-2

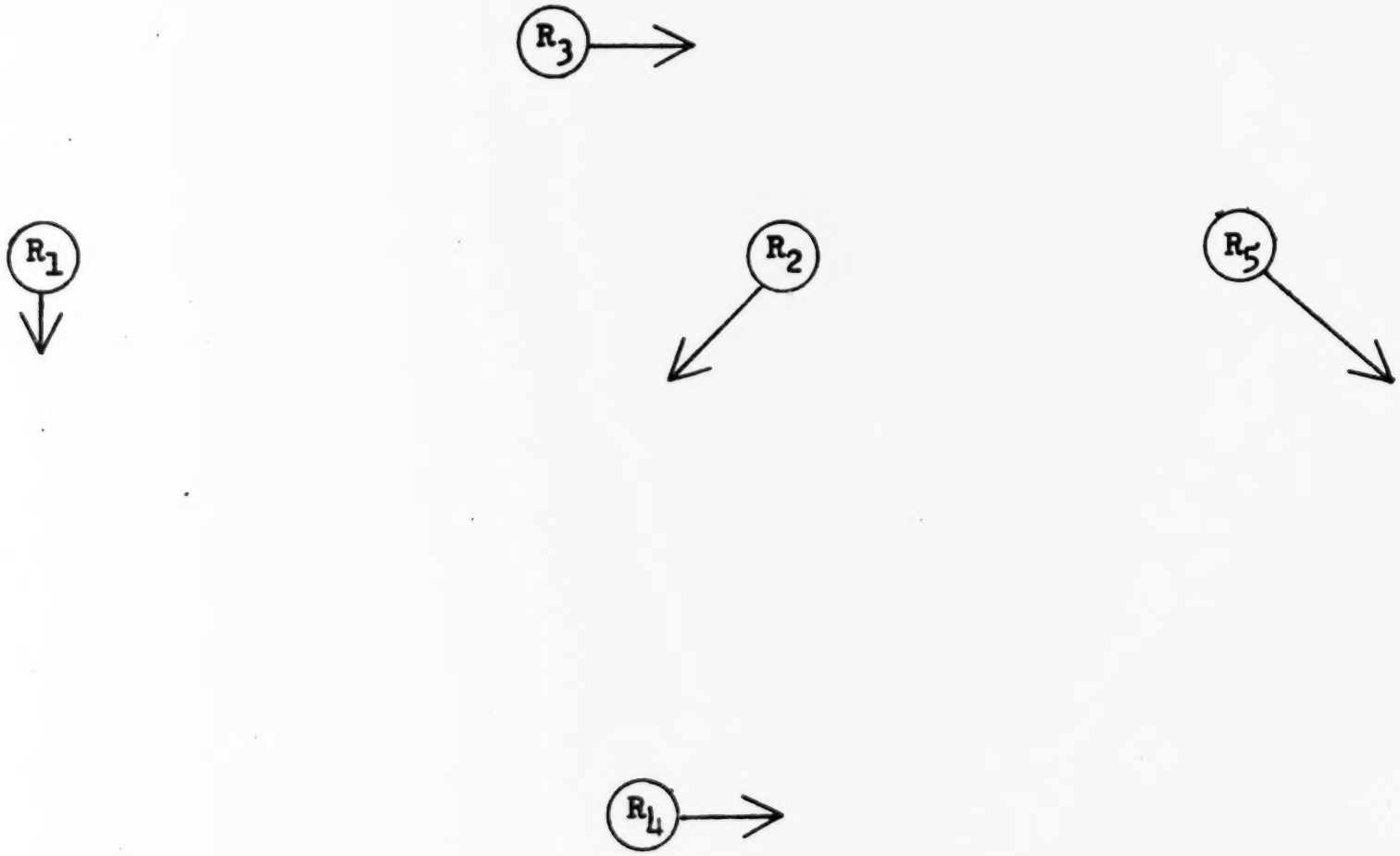
TRANSPARENCY 1-3



TRANSPARENCY 1-4



TRANSPARENCY 2-1



TRANSPARENCY 2-2

TABLE 2

Beta Studies for Path 145

Step	Comparisons								
	Direction 1-4-5					Direction 5-4-1			
	Other X_i	X_1	X_4	$d\beta$	I_β	$d\beta$	X_4	X_5	Other X_i
1		-1822	8720 9847	1127	275	4093	6186 1.0279	-4695	
2A	2 2	0104	6755 6666	0089	262	0340	8514 8274	0355	2 2
2B	3 3	-2706	7989 9093	1104	262	4194	4080 9074	-6250	3 3
3	2, 3 2, 3	0097	6728 6668	0060	444	0135	6159 6024	0200	2, 3 2, 3

TRANSPARENCY 3

GENERALIZATIONS

CAUSAL DIRECTION a-b-c	ACAUSAL DIRECTION c-b-a

Comparing beta differentials,

$$|\beta_{cb \cdot k} - \beta_{cb \cdot ka}| \quad < \quad |\beta_{ab \cdot k} - \beta_{ab \cdot kc}|$$

Then

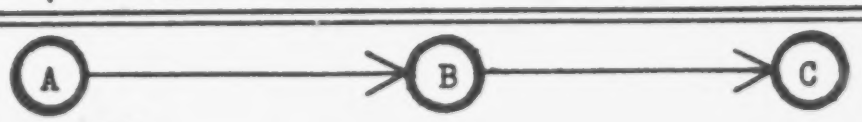
$$|\beta_{ca \cdot kb}| \quad < \quad |\beta_{ac \cdot kb}|$$

and also, it follows that

$$|r_{ab \cdot k}| \quad < \quad |r_{bc \cdot k}|$$

And finally,

$$|r_{ab \cdot kc}| \quad < \quad |r_{bc \cdot ka}|$$



Known 3-variable causal direction in a closed system with k other system variables operating.

Also, from logical analysis,

$$|r_{ac \cdot k'}| \quad < \quad |r_{ab \cdot k''}| \quad < \quad |r_{bc \cdot k'''}|$$

TRANSPARENCY 4